EXPERIMENTAL INVESTIGATION OF RIGID BODY ROCKING

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ABSTRACT

This paper presents the preliminary results of an experimental investigation on the behaviour of blocks under rigid body rocking. Nine blocks with different aspect ratios were tested with varying initial amplitudes and different materials at the contact interface. Three different materials, namely concrete, timber, and steel were used to construct the base on which the blocks could rock. The rocking characteristics of the blocks were compared to the predictions by Housner’s simple rocking model (SRM). Preliminary results show that the rocking response is strongly dependent upon the aspect ratio of the block, in general accordance to SRM. In addition, different materials at the contact interface play an essential role on the block’s rocking responses.

INTRODUCTION

Correctly designed and detailed reinforced concrete structures, under the prevailing capacity design concepts, are anticipated to exhibit inelastic response during a seismic event. However, this inelastic response leads to structural damage and permanent drift at conclusion of excitation which requires expensive post-earthquake repair intervention. Recent researches have shown that these permanent deformations can be minimised by implementing a self–centring system. One approach to get a self-centring system is to use post-tensioning masonry. For post-tension concrete masonry walls, a drift capacity of up to 1.5% was reached with little or no permanent deformations during dynamic tests [1] as well as static cyclic tests [2]. Another approach to get a self-centring system is to use post-tensioning masonry. The potentials of this system were highlighted in the U.S. PRESSS (PREcast Seismic Structural System) research program where a self-centring system when implemented with precast elements demonstrated superior seismic performance [3]. In this system, precast elements are connected using unbonded post-tensioning bars; the inelastic deformations are accommodated within the connection itself where a controlled rocking motion occurs (due to opening and closing of existing gap). Using this concept, the structural element remains elastic with limited damage. Recently [4], a drift capacity of 2.5% was reached by a self-centering precast reinforced concrete wall with approximately no damage in the specimen.

The advantages of the controlled rocking system reignited worldwide interest in adopting a damage-free strategy to structural aseismic design. Currently, extensive efforts worldwide exist to develop the “controlled-rocking” system; however there seems to be a conceptual issue with the basic rocking mechanics itself. To date, most of the available models deal with rocking according to Housner’s assumptions originated in the 1960s.

In order to better understand the basic mechanics of rigid body rocking, with the view to further develop this into a practical design, an extensive experimental research program is being conducted at the University of Auckland. This paper presents the preliminary results of a free rocking investigation. Nine blocks with nine different aspect ratios were tested in free rocking, activated by an initial rotation. In addition, the effects of different contact interface materials were studied by repeating the rocking experiments on three specifically constructed bases, made out of concrete, timber, and steel i.e. a total of twenty seven blocks were tested. The rocking characteristics of the blocks were compared with the prediction using Housner’s simple rocking model (SRM). Of course, seismically induced rocking behaviour is anticipated to be significantly more complex than that of free rocking. However, a good understanding of the free rocking response is deemed essential for the long term development of a model suitable for an arbitrary excitation.

LITERATURE REVIEW

In spite of the apparently simple nature of the rocking motion, its behaviour hides both a great richness in dynamic behaviour and a wide range of practical relevance. The rocking problem is stiff and highly nonlinear in nature. A study by Housner [5] has provided the basic understanding on the free rocking response of a rigid block. He analysed the oscillations of a rocking block and developed simple equations for rocking periods and energy dissipation. Later on, Lipscombe [6] experimentally showed that SRM was inaccurate for blocks with

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aspect ratios of less than four. In addition, for blocks with an aspect ratio of one the model incorrectly predicted the direction of rocking that the block underwent after impact. For squat blocks, bouncing effects and out-of-plane effects were significant. Such effects are not accounted for in SRM. Shenton and Jones [7] classified the motion of the block into five possible modes of response: rest, slide, rock, slide-rock, and free flight. Fielder et al. [8] experimentally examined the rocking response of in-plane asymmetrical blocks.

However, factors such as the materials of the base on which the block could rock and the block aspect ratio were rarely experimentally investigated.

**SIMPLE ROCKING MODEL (SRM)**

Following Housner [5], consider a symmetric block (Figure 1) with mass $m$, weight $W$, moment of inertia about the centre of rotation point $I_o$, and the location of centre of gravity a distance $h$ and $b$ from the block base and edge respectively. When the block is tilted an angle $\theta$ from the vertical, its equation of motion is then:

$$I_o \frac{d^2 \theta}{dt^2} = -WR \sin(\alpha - \theta) \quad \text{Eq (1)}$$

where $R = \sqrt{b^2 + h^2}$ and $\alpha$ is the angle that $R$ makes with the vertical.

![Figure 1: A rocking block.](image)

If the block is tilted an initial angle $\theta_0$ at time $t=0$, the time $T/4$ required by a block to fall from $\theta_0$ to $\theta = 0$ (Figure 2) is expressed as follows:

$$\frac{T}{4} = \left( \frac{I_o}{WR} \right) \cosh^{-1} \left( \frac{1}{1 - \theta_0 / \alpha} \right) \quad \text{Eq (2)}$$

Assuming that the impact of the block is inelastic (i.e. no bouncing) and using the concepts of conservation of angular momentum before and after impact, the reduction in the vibration energy of the block during rocking can be estimated by using the apparent coefficient of restitution $r$. In other words, the block angular velocity immediately after impact is $r$ times its angular velocity immediately before impact. The factor $r$ can be calculated as follows:

$$r = \left( 1 - \frac{mR^2}{I_o} \left( 1 - \cos 2\alpha \right) \right)^2 \quad \text{Eq (3)}$$

Using small angle approximation Housner solved Eq (1) and derived an expression for rocking amplitude. However, due to the reduction in kinetic energy with each impact, the amplitude of the vibration after $n$ impacts ($\theta_n$) decays. The amplitude $\theta_n$ can be expressed as follows:

$$\Phi_n = 1 - \sqrt{1-r^2[1-(\Phi_0)^2]} \quad \text{Eq (4)}$$

where $\Phi_n$ is the amplitude ratio after $n$ impacts $= \theta_n / \alpha$ and $\Phi_0$ is the initial amplitude ratio $= \theta_0 / \alpha$.

Finally, the successive half periods of the block will reduce. The half period after $n$ impacts (Figure 2) can be calculated as follows:

$$\frac{T_n}{2} = \frac{2}{\Phi_n} \left( \frac{I_o}{WR} \right) \tanh^{-1} \left( \sqrt{r^2[1-(\Phi_0)^2]} \right) \quad \text{Eq (5)}$$

More details about derivation of equations 1 to 5 could be found in [5].

![Figure 2: Free rocking oscillation.](image)
EXPERIMENTAL WORK

A series of carefully monitored free rocking experiments were conducted. The test set-up is shown in Figure 3. A typical block had a length and width of 780 and 190 mm, respectively. The block height ranged from 125 to 935 mm depending on the required aspect ratio. Nine aspect ratios (h/b) ranging between 0.7 and 4.9 were investigated. Figure 4 shows a block with an aspect ratio of 4.9 ready to test. A block consisted of a stack of solid bricks (designated 1017 solid) with two bricks positioned side by side in stake bond configuration (element c in Figure 3). The number of brick rows ranged from one to ten rows depending on the required height. These solid bricks were “sandwiched” between two steel plates (element a in Figure 3). The whole system was then tied together with six threaded rods passing through holes in the bricks and steel plates (element b in Figure 3). In such a way, the system rocked as a unit on the base. The lower steel plate was welded to two horizontal steel rods (15 mm diameter) to work as rollers for the block (element e in Figure 3). Hence, the impact surface with the base was minimised; therefore, all impacts became point impacts (as close as possible) and classic impact theory applies. Finally, in order to prevent sliding, one of the steel rollers was connected at each end to the end of a small steel plate (element d in Figure 3). This small steel plate was connected at its other end using pin joint to another steel plate. This last one was bolted to the foundation using steel bolts (Figure 5). In this way, the specimen was free to rock on its rollers but the pin joint of the steel plate prevented sliding of the specimen. The blocks rocked on a base which was fixed using steel bolts to a shaking table platform.

The displacement at the top of a block was measured by means of a horizontal direct-current displacement transducer (DCDT). Also, the displacement at the first or the second brick course (based on the tested aspect ratio) was measured using another DCDT as well as two portals (Figures 4 and 5). The strains in these portals, due to a specimen rocking, were measured using electrical strain gages. These strains were converted to rotation $\theta$. The measured horizontal displacements were converted to an angular rotation $\theta$. The DCDTs were mounted on a stiff steel post fixed to the shaking table platform. The portals were fixed directly to the same base as the rocking blocks. The data was recorded using a computer controlled data acquisition system with a frequency of 100 Hz.

Each block was initially tilted to an angle $\theta_o$ and then released with zero angular velocity at time $t = 0$. The test was repeated with different $\theta_o$ for each aspect ratio. Note that the initial rotation imparted to a block at the beginning of a free rocking test can not exceed $\alpha$ (i.e. $\Phi_o \leq 1$). Otherwise the block overturns. Hence, the maximum initial rotation has to be reduced as the slenderness of the block increases.

In order to study the effect of different interface materials on rocking behaviour, three different bases were used. The first base was a 10 mm thick steel plate. The second base was a 50 mm thick timber plate. The third base was a 75 mm thick steel plate. Note: different scales are used for sections A-A and B-B.

Figure 3: Typical test set-up [mm].

Figure 4: Specimens with 10 bricks high (h/b=5) ready to test.
The concrete plate was lightly reinforced. For each one of these bases, nine aspect ratios were tested (i.e., a total of twenty-seven blocks were tested).

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**RESULTS**

As mentioned, a total of twenty-seven blocks were tested. Each specimen was monitored using two DCDTs and two portal strain gages. In the next section, the effect of aspect ratio and interface material on rocking characteristics is presented. The rocking characteristics are presented in terms of $T_o$, $T_n$, $F_n$, and $r$ as defined previously.

**Tests on Concrete Base**

As an example of the obtained results, Figure 6 shows rocking vibration of two blocks with different aspect ratios: 3 and 5. Both blocks are tilted to approximately the same $F_o$ (approximately 0.51). As shown in the figure, both blocks initially have few oscillations of large amplitudes and slow rocking. With each impact, the amplitude decreases markedly corresponding with this amplitude reduction, and there is an increase in oscillation frequency. In addition, with increasing aspect ratio, the half period $T_n/2$ increases. This is obvious if we look to $T_o/4$ for both aspect ratios in the figure.

For high aspect ratio, the energy of vibration decreases slowly but for small aspect ratio, the energy decreases rapidly. With each impact, $T_n/2$ decreases. Figure 7 shows the reduction in $T_n/2$ for both blocks. Note that at $n=0$, $T_n/2$ was calculated by multiplying $T_o/4$ by 2. The half periods $T_n/2$ of the two blocks get closer after each impact. The half period after each impact was also calculated using Eq. (5) and presented in Figure 7. For an aspect ratio of 3, the prediction corresponded well to the experimental results. For an aspect ratio of 5, the equation predicted the half period at $n=0$ well; however, it overestimated the half periods of the other cycles. In addition, the decay rate of $T_n/2$ according to SRM was very slow compared to the experimental results.

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Another factor influencing the rocking response is the initial amplitude ratio $F_o$. Figure 9 shows the decay in $F_o$ for a block with an aspect ratio of 5. The block was released from rest with two different $F_o$: 0.30 and 0.50. As shown in the figure, for a
large initial amplitude ratio the energy of vibration decreases rapidly but for small amplitude oscillations the energy decreases slowly. The reduction in oscillation amplitude was also calculated using Eq (3). For both cases, the equation overestimated $\Phi_n$. Figure 8 shows the comparison between the experimental and SRM values.

\[ \Phi_n \]

Figure 8: Amplitude ratio ($\Phi_n$) subsequent to $n$-th impact for three aspect ratios (1, 3, and 5).

\[ \Phi_n \]

Figure 9: Amplitude ratio ($\Phi_n$) subsequent to $n$-th impact for one block ($h/b = 5$) with two different initial amplitude ratios $\Phi_0$ (0.3 and 0.5).

Figures 7 to 9 show that prediction of SRM for blocks with an aspect ratio of five is not accurate. Hence, a careful examination of the rocking behaviour of these blocks is required. A numerical simulation based on SRM and using $r$ corresponding to the actual geometry of a block (i.e Eq (3)) was carried out. The numerical implementation of SRM was based on an algorithm written in Matlab. Equation 1 was solved using Matlab’s ODE45 procedure. When a contact event was detected, Eq (3) was used to calculate the angular velocity after the impact. Figure 10 shows a comparison between the rocking vibration measured during the test and that calculated. Again, the SRM overestimated both the amplitude after impacts and the time required to dissipate the vibration energy. According to Eq (3), an $r$ value of 0.88 had been implemented in the numerical analysis. The best fit between the experimental values and the analysis happened by using $r$ values ranging from 0.72 to 0.79 (i.e. 82 to 90% of the value predicted by Eq (2)). Figure 11 shows the effect of $r$ value on oscillation amplitude ($\theta$). Note that no single value of $r$ completely satisfied the experimental data. Figures 10 and 11 show, also, the sensitivity of rocking behaviour to small changes. By reducing $r$ value by 10%, the time required to decay the vibration approximately drop to 50% of the time required by SRM (i.e. with the exact $r$). This is due to the high nonlinearity of the system where small variations in the damping (i.e. $r$ value) may alter the whole response of the block.

\[ \theta \]

One of the interested notes is that the energy dissipation of a rocking system according to SRM is only dependent on the block dimensions. However, in a pure rocking system the energy dissipated mainly due to radiation damping should be influenced by the material of the base (i.e. interface material) on which the block would rock. In the next paragraphs the effect of interface materials on rocking response.

Interface Material Effect
Three interface materials were investigated. Figure 12 shows the effect of interface material on rocking

\[ \text{Experimental} \quad \text{SRM} \]

Figure 10: Experimental vs. SRM oscillation amplitude ($\theta$).

\[ \text{r=0.79} \quad \text{r=0.72} \]

Figure 11: The effect of apparent coefficient of restitution ($r$) on oscillation amplitude ($\theta$).
vibrations for a block with aspect ratio of 5. As shown in the figure, the concrete base had the highest r followed by timber and steel bases. In addition, the vibration energy decreases very much rapidly for concrete base and slowly for steel base.

![Figure 12: Oscillation amplitude ratio for a block with h/b=5 and three different interface material.](image)

The effect of interface material on energy dissipation is also shown in Figure 13 in terms of $F_n$. The dissipation of vibration energy occurred most rapidly in the concrete base followed by timber and steel bases. However, for the first few impacts the steel base dissipated vibration energy faster than the timber base. On the same figure, $F_n$ calculated according to Eq (5) is plotted. In all cases, SRM overestimated $F_n$.

![Figure 13: Effect of interface material on $F_n$ for a block with h/b=5.](image)

The effect of interface materials on the rocking period is presented in terms of $T_o/4$ in Figure 14. For a block with an aspect ratio of 5, $T_o/4$ is plotted for different interface materials. As shown in the figure, the period is strongly dependent upon the initial amplitude ratio $F_o$. It is a nonlinear phenomenon; when $F_o$ is close to zero the period is short, and when $F_o$ is close to unity the period is long. In addition, the results obtained using the concrete base are consistent with only small variations. The period in case of concrete was slightly lower than other interface materials. The variations of the results were very high for timber and steel bases. In general, the effect of interface material on $T_o/4$ is insignificant. This shows that the significant effect of material interface on behaviour takes place following the first impact. $T_o/4$ was also calculated using Eq (2). The results are plotted in Figure 14. As shown in the figure, the results are in general agreement with the SRM except for high amplitude. For high $F_o$, the SRM gave $T_o/4$ three times the experimental values. Note that, when approaching a value of 1, Eq (2) becomes very sensitive to any small changes in $F_o$.

![Figure 14: Effect of interface material on free vibration period ($T$).](image)

**FINDINGS AND CONCLUSIONS**

This paper presents the preliminary results of an experimental investigation on free rocking behaviour of blocks. Nine blocks with different aspect ratios were tested with varying initial amplitudes and with different materials at the contact interface. Three different materials, namely concrete, timber, and steel were used to construct the base on which the blocks could rock. The rocking characteristics of the blocks were also calculated using a simple rocking model (SRM) based on Housner’s assumptions. The preliminary analysis of the results shows the following findings:

- For the same amplitude, a higher aspect ratio generated longer period and less vibration energy is dissipated.
- For the same aspect ratio, higher amplitude results in less vibration energy dissipation.
- For blocks with an aspect ratio of 5, SRM overestimated the number of impacts (n) that a block required before it come to rest. For small aspect ratio SRM underestimates n. For moderate aspect
ratios between 2 and 3, SRM predicts quite well.

- The apparent coefficient of restitution \( r \) is 82 to 90% of the values calculated based on SRM. Hence, the vibration energy dissipated faster than predicted using SRM. However, at this stage this finding cannot be generalized since it is based on few data. A more comprehensive analysis of the data is being carried out by the authors.

- The apparent coefficient of restitution \( r \) should take into considerations the interface material effect. For a concrete base, the energy of vibration decreased rapidly but for the other materials examined here (timber and steel) the energy decreased very slowly.

- The periods of the first quarter cycles (i.e. just before first impact) \( T_{1/4} \) for blocks that rocked on a concrete base were the lowest. For blocks that rocked on timber and steel, the vibrations in the results were very high. However, for the same amplitude, blocks that rocked on steel or a timber base had approximately the same period. The period of these blocks was slightly higher than for other blocks that rocked on a concrete base.

- \( T_{1/4} \) calculated using SRM are in general agreement with the measured values. However, for high amplitudes, SRM predicts much higher periods.

Based on these findings the following conclusions could be drawn:

- The free rocking response is strongly dependent upon the aspect ratio.
- The interface material plays an essential role in free rocking response.
- SRM should be revaluated and examined more carefully. In addition, development of SRM or an alternative model is required to accurately predict free rocking characteristics.

**REFERENCES**


